Fitting Zernike polynomials to ocular wavefront aberrations. Effect of the radial order on the fitting accuracy

Antonio J. Del Águila-Carrasco, MSc¹; José J. Esteve-Taboada, PhD¹; Noelia Martínez-Albert, OD¹; Daniel Monsálvez-Romín, MSc¹; Robert Montés-Micó, PhD¹

PURPOSE: To assess the accuracy of ocular wavefront aberration fits by means of Zernike reconstructions with different polynomial orders and different pupil sizes in normal healthy eyes.

SETTING: Experimental study performed at the University of Valencia Department of Optics.

METHODS: Ocular wavefront maps were obtained from 19 normal eyes using a Shack-Hartmann aberrometer. The tenth Zernike order fit was considered to be the real wavefront of the eyes. Different fittings from second to ninth Zernike orders were computed and compared with the tenth order. The ratio of the root mean square (RMS) error between the different fittings and the original wavefront was calculated, and a percentage value was given. This was performed for two pupil diameters: 3 and 6 mm.

RESULTS: The accuracy of the fitting improved when the Zernike polynomial order used to describe the wavefront increased. This was very evident for the 6 mm pupil size. In order to obtain a reproduction with 99% of the RMS error of the original wavefront, four orders were required for the small pupil size (3 mm), whereas nine Zernike orders were necessary for a 6 mm pupil.

CONCLUSIONS: The number of radial Zernike orders required to express ocular wavefront aberrations depends strongly on the pupil size. Four orders generally seem to be sufficient for small pupils, but for larger pupil sizes, more radial orders are needed if high accuracy is required.


Zernike polynomials have been widely used for describing ocular wavefront aberrations¹. These Zernike polynomials²,³ are defined within a circle (typically the pupil aperture of the eye). A given wavefront can be represented by a certain number of these polynomials, so the greater the number of polynomials, the better the fit to the real wavefront. Each one of these polynomials has a coefficient that multiplies them. This coefficient shows the magnitude of each mode; thus, a higher coefficient in a certain mode indicates that that mode is most important in the wavefront. Two different Zernike modes, one corresponding to the spherical defocus and one corresponding to a coma, are shown in Figure 1.

Zernike polynomials have been used to try to characterize the visual quality of the human eye⁴,⁵ and the optical quality of different corrections, like contact lenses⁶, intraocular lenses⁷,⁸ or refractive surgery⁹. Using Zernike polynomials is a fast, simple way of assessing the latter in normal eyes (with no diseases and no surgical records). However, describing pathological eyes (e.g. keratoconus) or eyes that have undergone refractive surgery,
such as laser in situ keratomileusis (LASIK) using the previously described polynomials can be a challenge, since corneal surfaces in these cases are highly irregular. In these situations, incorrect fits are often obtained, even when using a large number of Zernike modes\(^{10,11}\).

Another important issue with Zernike polynomials is that they are not orthogonal when measuring a discrete number of sample points, which is the usual situation. This basically means that the coefficients used for describing a particular wavefront will change depending on the number of modes considered for the description. The question that thus arises is to determine how many Zernike modes (or coefficients) are necessary to correctly reproduce the ocular wavefront for normal eyes. There will always be an error in the wavefront fitting, since there is no way of knowing the exact ocular wavefront, but the aim is to reduce that error as much as possible. Some studies have tried to answer the above-mentioned question, but only considering corneal aberrations\(^{12,13}\). To the best of our knowledge, there are no previous studies that have analyzed the goodness-of-prediction fit, taking into account the wavefront (or aberrations) of the whole eye.

The purpose of this study was to assess the ability of Zernike polynomials to reproduce ocular wavefronts of normal eyes, and to evaluate how many of these polynomials are required in order to have optimal reproducibility. The residual error of the wavefront fitting (root mean square, RMS) was analyzed using different numbers of Zernike modes.

### MATERIALS AND METHODS

Ocular wavefront aberrations were obtained using the crx1 adaptive optics visual simulator (Imagine Eyes, Orsay, France). This device measures aberrations of the eye with a Hart-Shackman (H-S) wavefront sensor (HASO3, Imagine Optic, France). The H-S sensor has an array of 32 x 32 microlenses, and a diode with a wavelength of 850 nm is used to illuminate the retina and capture the wavefront coming from the eye. The device has several interchangeable apertures that can be used to modify the pupil diameter in which the wavefront aberrations are obtained.

Wavefront aberrations were obtained for 19 eyes of 10 participants. This study had Institutional Review Board approval and the research adhered to the tenets of the Declaration of Helsinki. The participants who took part in this study were young (between 21 and 30 years old), healthy subjects. They had no known ocular conditions, and were not taking any ocular or systemic medication. The participants’ spherical and cylindrical error was always below ±0.5 D.

Three wavefront aberration maps were obtained and then averaged for each subject’s eye, using a pupil diameter of 3 and 6 mm. Since the real wavefront of the eye is not known, the tenth-order Zernike polynomial fit was considered to be the exact wavefront of each eye. This wavefront was then fitted for different Zernike orders (from second to ninth order) and compared to the tenth order fit. To that end, the RMS error was calculated for each fit. Finally, the percentage defined by each fit of the original wavefront considered was calculated in order to determine the accuracy of the different wavefront fits with respect to the tenth order fit.
RESULTS

The lower the order of Zernike terms used for fitting, the lower the percentage of RMS error defined by that fit as regards what is considered the real wavefront (tenth-order fit). This means that when a higher number of polynomials is taken into consideration, the fit is better and more similar to the real one. The relationship between the number of Zernike orders used and the percentage of RMS error defined by each fit is non-linear. Figure 2 shows the percentage of RMS error defined by each fit from second to tenth order, and for two different pupil sizes (3 and 6 mm diameter). Error bars represent the standard deviation for all the measurements. To define 95% of the original wavefront, two orders are enough in a 3-mm pupil; however seven orders are required when the pupil diameter under study is 6 mm. For a 3-mm pupil, only four orders were required to reproduce 99% of the original wavefront, whereas nine orders were needed in order to reach this value when the analyzed pupil had a diameter of 6 mm. Clearly, when the pupil size is increased, fitting the wavefront using a very low number of Zernike orders results in very poor reproducibility.

Figure 3 shows what we considered the real wavefront (fitted with Zernike polynomials up to tenth order) from subject #4 in comparison to one fit using three Zernike orders. It is evident that when only three radial orders are used for the wavefront fitting, the surface is smoother than the wavefront fitted with ten orders, and therefore, it will be less accurate as well.

DISCUSSION

The use of Zernike polynomials for describing wavefront aberrations has been very popular recently. Nevertheless, there are some cases (very irregular corneas, for example) where using Zernike fitting is not the best way to proceed\textsuperscript{10}, because of the lack of accuracy in the outcome. The issue is trying to define which order should be reached for describing ocular aberrations. It is clear that for compensating typical refractive errors (sphere, cylinder and prism), no further than order two (or Low Order Aberrations [LOAs]) should be analyzed. Subjects can reach very acceptable vision by only compensating LOAs. The problem arises when the requirements are higher; for example, in refractive
corneal surgery\textsuperscript{9}, where it is desirable not to add Higher Order Aberrations (HOAs) to the patient, an optimal reproduction of the wavefront is required, which translates into the use of a greater number of Zernike polynomials. What then is the right number of orders that should be considered for wavefront aberration reproduction? The answer, obviously, depends on the requirements, as stated above. It must be taken into account though, that as the pupil size increases, more Zernike polynomials are needed in order to have a good fit of the real wavefront, even with normal healthy eyes, like the ones in this study. Thus, the optimal number of Zernike orders necessary to describe an ocular wavefront will depend on the irregularity of the cornea or any other eye media, the pupil size and the task that is to be performed using that wavefront. For example, trying to represent the ocular wavefront of an eye with a highly irregular cornea using only Zernike polynomials up to second order will yield a very poor result. This is because the fitted surface will be overly smooth and inaccurate when comparing it to the real wavefront.

A study similar to this one was performed in several groups of participants, but only analyzing corneal aberrations\textsuperscript{12}. Subjects who underwent corneal surgery (LASIK, photorefractive keratoplasty, and radial keratotomy) and subjects with different degrees of keratoconus, among others, were included. All subjects needed a greater number of Zernike polynomials to better fit the corneal surface than the subjects with normal healthy eyes. A similar tendency was also found with the pupil size, which is consistent with the fact that when the wavefront is greater in extension (or in other words, the pupil is bigger), the aberrations increase.

Bearing in mind the above, special care should be taken when describing ocular wavefront aberrations. If the pupil size is small (around 3-mm diameter), fitting the wavefront with Zernike polynomials up to fourth order can be accurate enough for some purposes, but if a greater pupil size is required, then the fit should be performed with a higher number of Zernike polynomials, depending on the purpose and accuracy needed. For example, in this study, for 6-mm pupils, nine Zernike orders are necessary for a high accuracy fitting (99% of the real wavefront defined) of the wavefront aberrations of normal healthy eyes.

In conclusion, the accuracy of Zernike fitting of ocular wavefront aberrations in normal eyes depends on the number of radial orders used in the fit. The higher the number of orders used, the more accurate the fit. This is more evident when the pupil size is larger, when it becomes crucial to use a high number of Zernike polynomials to obtain a proper and accurate fit.

REFERENCES